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Failure criteria for isotropic materials, applications to low-density types ☆

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Abstract

Isotropic failure criteria are derived in a general form that applies to both homogeneous materials and to porous, low-density materials. Specific applications are made to closed-cell foam materials, with the properties type parameters determined from experimental data. It is found that in addition to the usual yielding type of failure behavior, a fracture type behavior can arise under certain tensile stress conditions and must be taken into explicit account in forming the failure criteria for low-density materials. Published by Elsevier Science Ltd.

Keywords: Failure criteria; Low-density materials; Closed-cell foam

1. Introduction

The yielding and failure of materials is a topic of great current interest, as well as of considerable historical interest. The common criteria still in wide usage are the Mises and Tresca forms. While these forms are quite satisfactory for the initial yielding of ductile metals, they are completely inadequate for just about any other purpose. The present work considers appropriate failure criteria forms for a wide variety of isotropic materials ranging from those for ductile, full-density materials to those for very porous, but still isotropic, low-density materials.

The term "failure criteria" will be used in a broad and inclusive sense as relating to anything from sudden, catastrophic failure due to fracture for brittle materials, to the inception of yielding in the more ductile materials. A more descriptive terminology would be to refer to the present subject as that of yielding or failure, where appropriate, but for simplicity of reference, it will be condensed to just "failure" or failure

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criteria. In perspective, the concern here is with characterizing the spectrum of isotropic material behaviors at the threshold of significant deviation from the linear stress–strain condition.

There has been much effort expended on material failure criteria over a very long period of time. Some historical aspects of the subject for full-density materials were included with work given by Christensen (1997) and need not be repeated here. For a general historical summary see Paul (1968). There likely will be much more work done on the general topic due to its primal importance for applications. In the present work, consideration will be given to both full-density and low-density materials types, but much of the applicational emphasis will be directed toward the newer forms of low-density materials.

For low-density materials the most widely referenced work is that of Gibson and Ashby (1997). The major emphasis in their work is for open-cell materials and the proposed failure criteria require some properties of the cell wall material. Different mechanism-based criteria are proposed for various failure modes such as plastic collapse, elastic buckling, and brittle fracture. For the particular case of elastic buckling, no analytical expression and formulae have been proposed to fit the theoretical buckling surface (Puso and Govindjee, 1995; Zhang et al., 1997). More recently, quadratic models for the yield surfaces of metallic foams have been proposed by Deshpande and Fleck (2000).

2. Polynomial failure criteria

As the first mathematical step toward failure criteria for isotropic materials, take a polynomial expansion in the invariants of the stress tensor, σ_{ij}, and truncate the expansion at terms of second degree, similar to that for strain energy, giving

$$\Delta\sigma_{ii} + \beta\sigma_{ii}^2 + \gamma\sigma_{ij}\sigma_{ij} \leqslant 1. \tag{1}$$

The three resulting materials parameters, Δ, β, γ are to be determined from failure data for any particular material of interest.

Take σ_{11}^{T}, σ_{11}^{C}, σ_{12}^{Y} as being the experimentally determined failure (or yield) stresses for uniaxial tension and compression, and for simple shear, respectively. Then Eq. (1) can easily be shown to take the explicit form

$$\left(\frac{1}{\sigma_{11}^{\mathrm{T}}} - \frac{1}{|\sigma_{11}^{\mathrm{C}}|}\right)\sigma_{ii} + \left[\frac{1}{\sigma_{11}^{\mathrm{T}}|\sigma_{11}^{\mathrm{C}}|} - \frac{1}{2(\sigma_{12}^{\mathrm{Y}})^2}\right]\sigma_{ii}^2 + \frac{\sigma_{ij}\sigma_{ij}}{2(\sigma_{12}^{\mathrm{Y}})^2} \leqslant 1, \tag{2}$$

where $|\sigma_{11}^{\mathrm{C}}|$ is the magnitude of σ_{11}^{C}.

It will be useful to have Eq. (2) expressed in terms of deviatoric stress, s_{ij}, then

$$\left(\frac{1}{\sigma_{11}^{\mathrm{T}}} - \frac{1}{|\sigma_{11}^{\mathrm{C}}|}\right)\sigma_{ii} + \left[\frac{1}{\sigma_{11}^{\mathrm{T}}|\sigma_{11}^{\mathrm{C}}|} - \frac{1}{3(\sigma_{12}^{\mathrm{Y}})^2}\right]\sigma_{ii}^2 + \frac{s_{ij}s_{ij}}{2(\sigma_{12}^{\mathrm{Y}})^2} \leqslant 1, \tag{3}$$

where

$$s_{ij} = \sigma_{ij} - \frac{\delta_{ij}}{3}\sigma_{kk}.$$

Any of the three alternative relations (1), (2), or (3) comprise the most general second degree polynomial form for use with isotropic materials. Form (1) and its many simple and similar variations has been used for fitting materials data over a time span far predating any contemporary activities. Forms (2) and (3), stated here, probably are the most transparently clear and direct possible types and both of them will be employed or partially employed here. It will be very helpful to compare and contrast full-density material behavior with that of low-density types, thus the former class are treated next.

3. Full density, homogeneous materials

Attention is now restricted to full-density materials such as most metals, ceramics and polymers which ordinarily have at most a minor degree of porosity. These materials may have other "filler" phases so long as the material remains essentially homogeneous and isotropic at an acceptable macro-scale.

For full-density materials of the type just prescribed, they always show the physical characteristic that the failure stress in uniaxial tension is never greater than the magnitude of that in uniaxial compression. Accordingly, take the restriction that

$$\sigma_{11}^{T} \leqslant |\sigma_{11}^{C}|, \tag{4}$$

for the materials of study in this section.

Take a hypothesis that full-density materials do not fail under hydrostatic compressive stress but always fail under sufficiently large hydrostatic tension. With this condition and for restriction (4), the coefficient of σ_{ii}^2 in failure criterion (3) must vanish, giving

$$\sigma_{12}^{Y} = \sqrt{\frac{\sigma_{11}^{T}|\sigma_{11}^{C}|}{3}}. \tag{5}$$

With relation (5) the polynomial failure criterion (3) is left as

$$\left(\frac{1}{\sigma_{11}^{T}} - \frac{1}{|\sigma_{11}^{C}|}\right)\sigma_{ii} + \left(\frac{3}{2}\right)\frac{s_{ij}s_{ij}}{\sigma_{11}^{T}|\sigma_{11}^{C}|} \leqslant 1, \tag{6a}$$

which is now a two parameter form involving only σ_{11}^{T} and σ_{11}^{C}.

Writing out Eq. (6a) in component form gives

$$\left(\frac{1}{\sigma_{11}^{T}} - \frac{1}{|\sigma_{11}^{C}|}\right)(\sigma_{11} + \sigma_{22} + \sigma_{33}) + \frac{1}{2\sigma_{11}^{T}|\sigma_{11}^{C}|}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2)] \leqslant 1. \tag{6b}$$

When taken in terms of principal stresses, the shear stress terms in Eq. (6b) disappear, which is the form used by Raghava et al. (1973). Relation (6a) shows a remarkable simplicity considering the complexity and range of physical effects that it contains, as will be seen in the following developments.

As shown by Christensen (1997) relation (6a) can be written in alternative form as

$$\frac{\alpha k}{\sqrt{3}}\sigma_{ii} + \frac{1}{2}(1 + \alpha)s_{ij}s_{ij} \leqslant k^2, \tag{7}$$

where

$$k = \frac{|\sigma_{11}^{C}|}{\sqrt{3}}, \qquad \alpha = \frac{|\sigma_{11}^{C}|}{\sigma_{11}^{T}} - 1, \quad \alpha \geqslant 0. \tag{8}$$

The equivalence of Eqs. (6a) and (7) can be verified by direct substitution from Eq. (8). The advantage of form (7) over that of Eq. (6a) is that only one parameter need be varied in order to scan the whole range of different types of materials behavior, that being the non-dimensional parameter, α. Parameter k merely changes the scale.

For very ductile behavior the yield stresses in uniaxial tension and compression are the same, $\sigma_{11}^{T} = |\sigma_{11}^{C}|$. With $\alpha = 0$ from Eq. (8), then Eq. (7) gives the Mises criterion as the descriptor of the yielding of such ductile materials.

For the brittle range of behavior, take behavior at the opposite end of the scale from that of the ductility condition stated above, thus

$$\sigma_{11}^{\mathrm{T}} \ll |\sigma_{11}^{\mathrm{C}}|, \quad \text{giving from Eq. (8) } \alpha \gg 1. \tag{9}$$

It will be of general interest as well as be helpful in the low-density material applications to follow to further examine brittle behavior for full-density materials. Thus conditions (9) apply from this point on in this section. Other aspects of behavior are treated at some length in Christensen (1997).

For the brittle range with large α (Eq. (9)), take an expansion of stress in powers of α as

$$\sigma_{ij} = \sigma_{ij}^{(0)} + \frac{\sigma_{ij}^{(1)}}{\alpha} + \frac{\sigma_{ij}^{(2)}}{\alpha^2} + \cdots. \tag{10}$$

Then

$$\sigma_{ii} = \sigma_{ii}^{(0)} + \frac{\sigma_{ii}^{(1)}}{\alpha} + \cdots, \quad \text{and} \quad s_{ij} = s_{ij}^{(0)} + \frac{s_{ij}^{(1)}}{\alpha} + \cdots. \tag{11}$$

Substitute Eq. (11) into Eq. (7) and collect terms involving like powers of α, to give

$$\alpha\left[\frac{k}{\sqrt{3}}\sigma_{ii}^{0} + \frac{1}{2}s_{ij}^{(0)}s_{ij}^{(0)}\right] + \left[\frac{k}{\sqrt{3}}\sigma_{ii}^{(1)} + \frac{1}{2}s_{ij}^{(0)}s_{ij}^{(0)} + s_{ij}^{(0)}s_{ij}^{(1)} - k^2\right] + \mathrm{O}\left(\frac{1}{\alpha}\right) \leqslant 0. \tag{12}$$

To proceed further there must be considered two separate cases controlled by mean normal stress

$$\text{(i) } \sigma_{ii} < 0 \quad \text{Hydrostatic pressure,} \quad \text{or} \quad \text{(ii) } \sigma_{ii} > 0 \quad \text{Hydrostatic tension.} \tag{13}$$

But before investigating the two cases in Eq. (13) note that for $\sigma_{ii} = 0$ relation (7) directly gives $s_{ij} \to 0$ for very large α, the extreme brittle state.

Case (i) $\sigma_{ii} < 0$:

Begin by taking $\sigma_{ii}^{(0)} < 0$, then the first term in Eq. (12) gives

$$\frac{k}{\sqrt{3}}\sigma_{ii}^{(0)} + \frac{1}{2}s_{ij}^{(0)}s_{ij}^{(0)} \leqslant 0, \tag{14}$$

where all other terms in Eq. (12) have been neglected for large α. Thus Eq. (14), using Eq. (8), becomes the final form

$$\frac{1}{2}s_{ij}s_{ij} \leqslant -|\sigma_{11}^{\mathrm{C}}|\frac{\sigma_{ii}}{3} \quad \text{for } \sigma_{ii} < 0, \quad \sigma_{11}^{\mathrm{T}} \ll |\sigma_{11}^{\mathrm{C}}|. \tag{15}$$

For the specific example of simple shear stress, τ, under pressure, p, Eq. (15) becomes

$$\tau^2 \leqslant |\sigma_{11}^{\mathrm{C}}|p.$$

Relation (15) is descriptive of behavior for a material that is so damaged it is in effect cohesionless or almost cohesionless, but it can be made to behave like an integral cohesive material through the application of hydrostatic pressure.

Case (ii) $\sigma_{ii} > 0$:

Now try the case of

$$\sigma_{ii}^{(0)} \geqslant 0. \tag{16}$$

Retaining only the first term in Eq. (12) for large α gives Eq. (14) again, but with Eq. (16) it can only be satisfied by

$$\begin{aligned} \sigma_{ii}^{(0)} &= 0 \\ s_{ij}^{(0)} &= 0. \end{aligned} \tag{17}$$

Thus the expansion (12) in powers of α must begin with the second term (the first term vanishes) and retaining only this second term in Eq. (12), using Eq. (17), gives

$$\left[\frac{k}{\sqrt{3}}\sigma_{ii}^{(1)} + 0 + 0 - k^2\right] \leqslant 0, \tag{18}$$

or

$$\frac{\sigma_{ii}^{(1)}}{\sqrt{3}} \leqslant k. \tag{19}$$

With all other higher order terms in α neglected, Eq. (19) with Eqs. (11) and (17) becomes

$$\sigma_{ii} \leqslant \frac{\sqrt{3}k}{\alpha}. \tag{20}$$

Using k and α from Eq. (8) with Eq. (9) then gives

$$\sigma_{ii} \leqslant \sigma_{11}^{\mathrm{T}} \quad \text{for } \sigma_{ii} > 0, \quad \sigma_{11}^{\mathrm{T}} \ll |\sigma_{11}^{\mathrm{C}}|. \tag{21}$$

Thus in the present context, the mean normal stress governs all aspects of very brittle "tensile" failure behavior.

Relations (15) and (21) will be flagged as denoting brittle behavior. However, it should be recognized that there is a whole range of brittle behavior for full-density materials (Christensen, 2000). Most of this brittle behavior range is not characterized by Eqs. (15) and (21) because as seen from the derivation of these two relations, they actually exist only at the limit or the extreme of the brittle range. Relation (21) also will be called brittle fracture when its counterpart arises in the context of low-density material behavior, considered next.

4. Low-density materials

Common usage of the term low-density materials usually means either closed-cell or open-cell forms at a rather large or even extreme level of porosity. For these materials there does not appear to be any possible direct simplification such as was the case for that of full-density materials not exhibiting failure under hydrostatic pressure. Nor would the "mirror image" of this simplification be plausible. That is, independence of hydrostatic tensile stress type failure is not justifiable for low-density materials. In fact, these types of materials exhibit a type of brittle fracture behavior under tension as shown schematically in Fig. 1 for a stress state involving uniaxial and shear stress.

The cutoff behavior shown in Fig. 1 would imply no interaction between modes I and II if this cutoff behavior were to be described by classical fracture mechanics. Alternatively, the brittle fracture criterion derived in the preceding section could be used. Following the latter course take

$$\sigma_{ii} \leqslant \hat{\sigma}_{11}^{\mathrm{T}}, \tag{22}$$

where the form of Eq. (21) is repeated here but without the restrictions that must accompany Eq. (21) in the full-density materials context. While form (22) is motivated by form (21) for full-density materials, it is of more general form as assumed and used here in the context of low-density materials.

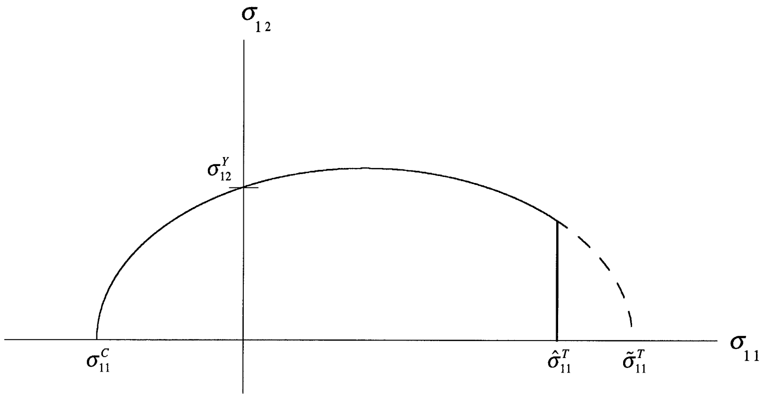

Fig. 1. Failure surface for low-density material under combined shear and normal stresses with a tensile cutoff for brittle fracture.

As one other observation, typically for low-density materials $|\sigma_{11}^{\mathrm{C}}| \leqslant \hat{\sigma}_{11}^{\mathrm{T}}$, referring to Fig. 1, but it is not necessary to assume this condition in the following forms.

Take the non-cutoff part of the behavior in Fig. 1 in the form of the polynomial criterion (2) as

$$\left(\frac{1}{\tilde{\sigma}_{11}^{\mathrm{T}}}-\frac{1}{|\sigma_{11}^{\mathrm{C}}|}\right)\sigma_{ii}+\left[\frac{1}{\tilde{\sigma}_{11}^{\mathrm{T}}|\sigma_{11}^{\mathrm{C}}|}-\frac{1}{2(\sigma_{12}^{\mathrm{Y}})^2}\right]\sigma_{ii}^2+\frac{\sigma_{ij}\sigma_{ij}}{2(\sigma_{12}^{\mathrm{Y}})^2}\leqslant 1. \tag{23}$$

In Eq. (23) $\tilde{\sigma}_{11}^{\mathrm{T}}$ is the "phantom" tensile strength that would exist if there were no brittle fracture cutoff behavior, as in Fig. 1. Note that in general $\tilde{\sigma}_{11}^{\mathrm{T}}$ can be larger or smaller than $\hat{\sigma}_{11}^{\mathrm{T}}$, Fig. 1. In other words for $\tilde{\sigma}_{11}^{\mathrm{T}} < \hat{\sigma}_{11}^{\mathrm{T}}$ there could be a brittle fracture cutoff behavior in other stress states such as biaxial tension, that does not show up in the stress state depicted in Fig. 1.

It is now seen that characterizing the failure of low-density materials is quite complex, certainly more complex than that for full-density materials at the same level of treatment. In general, four parameters are required in the characterization given by Eqs. (22) and (23). At the best, with no intervention by a brittle fracture effect, three parameters are still required for Eq. (23) to be used. Only in the very special case when a relation of type (5) form could be justified could one then eliminate one further parameter, reducing the possible theory to a two-parameter form. However, this latter course will not be followed here, and the applications to be given next will of necessity involve the full four-parameter form.

The most important observation to be made at this point is as follows. As seen in the last section for full-density materials, brittle fracture behavior arises as a natural, self-contained aspect of the polynomial failure criterion. The situation with low-density materials is quite the opposite and much more involved. The brittle fracture effect does arise here, but in effect it is in competition with the polynomial failure criterion "modes" of failure and it must be imposed as a separate condition. This fundamental difference of behavior for the two materials types is the key to treating both cases.

5. Specific stress states

First, consider the stress state depicted in Fig. 1, namely uniaxial and shear stresses in interaction. For uniaxial stress, σ_{11}, and shear stress, σ_{12}, relation (23) becomes

$$\left(\frac{1}{\tilde{\sigma}_{11}^{\mathrm{T}}}-\frac{1}{|\sigma_{11}^{\mathrm{C}}|}\right)\sigma_{11}+\frac{\sigma_{11}^2}{\tilde{\sigma}_{11}^{\mathrm{T}}|\sigma_{11}^{\mathrm{C}}|}+\frac{\sigma_{12}^2}{(\sigma_{12}^{\mathrm{Y}})^2}\leqslant 1. \tag{24}$$

The brittle fracture, tension cutoff form (22) is simply

$$\sigma_{11} \leqslant \hat{\sigma}_{11}^{\mathrm{T}}. \tag{25}$$

Before considering a specific data example, another stress state of interest should be stated, that of biaxial stresses σ_{11} and σ_{22}, with all other stresses vanishing. Relation (23) in this case becomes

$$\left(\frac{1}{\tilde{\sigma}_{11}^{\mathrm{T}}} - \frac{1}{|\sigma_{11}^{\mathrm{C}}|}\right)(\sigma_{11} + \sigma_{22}) + \frac{1}{\tilde{\sigma}_{11}^{\mathrm{T}}|\sigma_{11}^{\mathrm{C}}|}(\sigma_{11} + \sigma_{22})^2 - \frac{\sigma_{11}\sigma_{22}}{(\sigma_{12}^{\mathrm{Y}})^2} \leqslant 1. \tag{26}$$

The brittle fracture, tension cutoff form (22) becomes

$$\sigma_{11} + \sigma_{22} \leqslant \hat{\sigma}_{11}^{\mathrm{T}}. \tag{27}$$

6. Experimental investigation of closed-cell foams

6.1. Material and test specimens

To examine the proposed yield criteria described by Eqs. (22) and (23), the failure of a rigid, closed-cell foam under biaxial stress states was measured. The cellular material was a polyvinylchloride foam (Divinycell H200, DIAB International AB, Sweden) with a nominal density of 200 kg/m^3. This particular class of closed-cell foam is widely used in marine applications. It is normally produced in slab form for sandwich construction and in this application the most important properties are those normal to the slab surface. These are the properties in the so-called "rise" direction. Therefore, biaxial testing was performed for the combination of through-thickness normal stress (tension or compression) and shear stress. Specimens having a hollow cylindrical gage section and square ends were machined from the slabs. The gage section of the specimen was 25.4 mm long and had an outer diameter of 21.0 mm and an inner diameter of 15.9 mm. These dimensions do not allow a thin-walled cylinder approximation for shear stress. However, all specimens exhibited yielding behavior and to calculate shear stress at failure, the following relation for a fully plastic hollow cylinder was used

$$\sigma_{12}^{\mathrm{Y}} = \frac{3T}{2\pi(r_{\mathrm{o}}^3 - r_{\mathrm{i}}^3)}, \tag{28}$$

where T is the torque at yield, and r_{o} and r_{i} are the outer and inner radii of the specimen gage section, respectively. It should be noted that difference in shear strength due to assuming completely elastic versus completely plastic behavior at failure is only 12%.

6.2. Test method

Specimens were tested in an axial–torsion biaxial test machine (MTS Systems Corp., Model 858, Eden Prairie, MN). Tests were performed along two load paths: (1) by first applying a constant normal stress (force) and then shearing the specimen to failure in torsion and (2) by first applying a constant shear stress (torque) and then deforming the specimen axially to failure. Axial strain was measured using a commercial extensometer (MTS, Model 2620-524) and shear strain was measured using a rotation gage built specifically for the torsion experiments (John A. Shepic, Lakewood, CO). It was found necessary to cover the surface of the foam specimen with adhesive-backed copper tape for the purpose of preventing slippage of the displacement and rotation transducers. Preliminary tests showed that the tape had no effect on the failure stress or location for any stress state. All tests were conducted in displacement or rotation control at rates

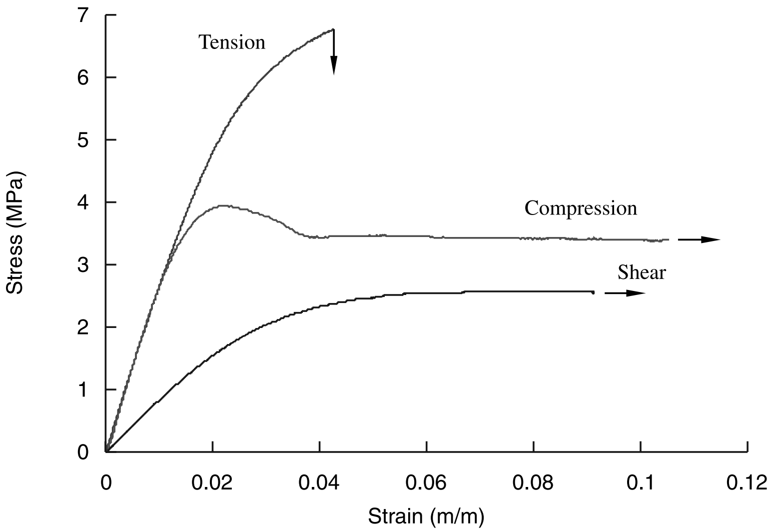

Fig. 2. Uniaxial stress–strain response of Divinycell H200 in tension, compression, and shear.

that yielded an initial strain rate of $1E-4\ s^{-1}$. In the biaxial tests, the force and torque were controlled to maintain a constant normal or shear stress, depending on the load path taken.

6.3. Experimental results

Representative stress–strain responses for the three types of uniaxial stress experiments are plotted in Fig. 2. The ductility of the foam in compression and shear is obvious and experiments were stopped prior to catastrophic failure. In both cases yielding occurred due to collapse of the cell structure. Typical failed specimens are shown in Fig. 3. For these tests, the yield strength was taken to be the maximum value of stress, which for shear was the flow stress and for compression was the peak stress. Although some evidence of yielding occurred in uniaxial tension, failure occurred in a more brittle fashion at a strain of about 4%.

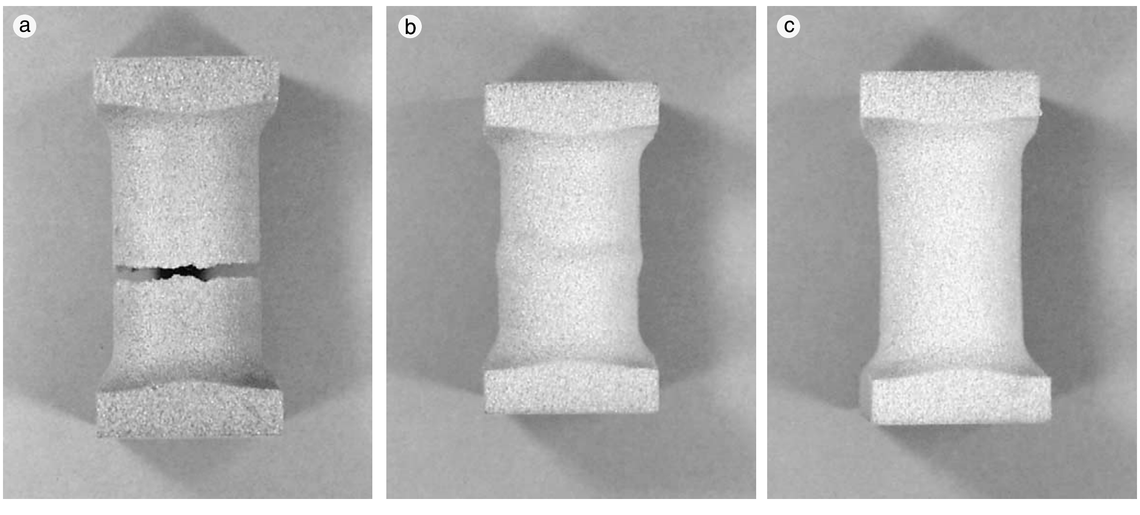

Fig. 3. Failure modes for Divinycell H200 in uniaxial (a) tension, (b) compression, and (c) shear (torsion).

Table 1
Uniaxial properties of Divinycell H200 closed-cell foam in rise direction

Loading type	Modulus (MPa)	Strength (MPa)	Elongation at break (%)
Tension	260	6.93	4.3
Compression	280	3.55	–
Shear	96	2.40	–

All tensile fracture surfaces were at 90° to the load axis as shown in Fig. 2. For tension tests, the yield strength was taken to be the (peak) value of stress at rupture. Properties obtained from the uniaxial tests are summarized in Table 1.

Yield strengths under biaxial stress states are plotted in Fig. 4 along with a fit of the three-parameter polynomial criterion (24). The material response was ductile for combinations of compression and shear, but transitioned from ductile to brittle for states of tension plus shear. This transition occurred at a tensile stress of approximately 4 MPa, which is also shown in Fig. 4. The tensile cutoff for brittle failure is plotted, which was simply obtained from the average of several uniaxial tension tests. A reasonably good fit is obtained to the biaxial data, but only when both the proposed three-parameter polynomial criterion and a tensile cutoff are used.

The fracture planes of specimens that failed in a brittle fashion under combined tension and shear were oriented at an oblique angle. Furthermore, these surfaces were not coincident with the planes of maximum principal tensile stress. While this might suggest that failure is not controlled by maximum normal stress, it is not clear that this observation alone is sufficient to distinguish between the two tensile cut-off failure conditions.

As discussed earlier, the combination of shear and uniaxial stress does not probe all cases of tensile failure. To demonstrate this, a plot of the shear/uniaxial failure data in principal stress space is given in Fig. 5. Also shown are the hypothetical fracture envelopes for tensile failure controlled by maximum normal stress and mean hydrostatic tension as specified by the condition (25). The distinction between these two tensile fracture criteria is not significant in the stress regime covered by the experimental data. However, the predictions based on the two cutoff conditions are distinguishable for the particular case of equal biaxial tension. These tests are planned for future studies of closed-cell foams.

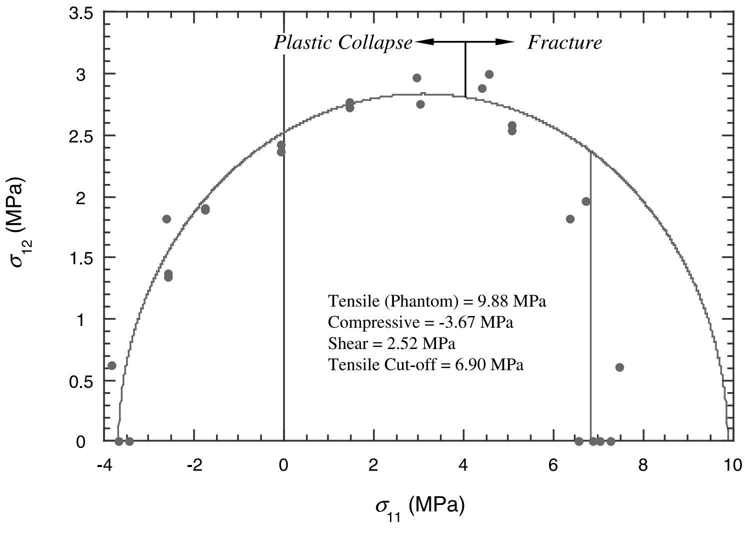

Fig. 4. Comparison of yield data for Divinycell H200 closed-cell foam with three-parameter yield and tensile cutoff failure criteria.

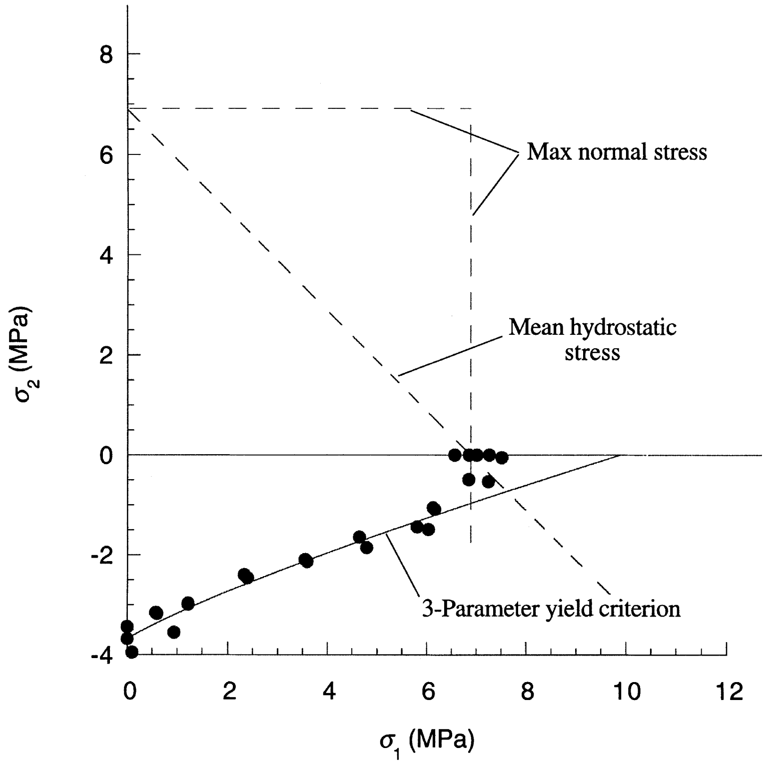

Fig. 5. Yield data for Divinycell H200 closed-cell foam in principal stress space.

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